

**Hong Kong Mathematics Olympiad (2015/16)**  
**Heat Event (Group)**  
**香港數學競賽 (2015/16)**  
**初賽項目(團體)**

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 最初甲瓶裝有 1 公升酒精，乙瓶是空的。

第 1 次將甲瓶全部的酒精倒入乙瓶中，第 2 次將乙瓶酒精的  $\frac{1}{2}$  倒回甲瓶，

第 3 次將甲瓶酒精的  $\frac{1}{3}$  倒入乙瓶，第 4 次將乙瓶酒精的  $\frac{1}{4}$  倒回甲瓶，……，

第 2016 次後，甲瓶還有多少公升酒精？

At the beginning, there was 1 litre of alcohol in Bottle A and bottle B is an empty bottle.

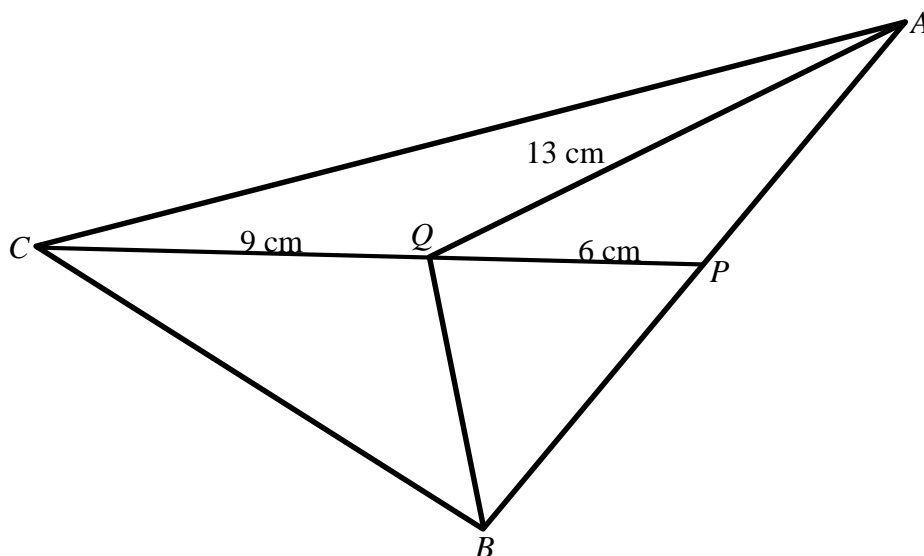
First, pour all alcohol from bottle A to bottle B; second, pour  $\frac{1}{2}$  of the alcohol from bottle B back to

bottle A; third, pour  $\frac{1}{3}$  of the alcohol from bottle A to bottle B; fourth, pour  $\frac{1}{4}$  of the alcohol from bottle

B back to bottle A, ... . After the 2016<sup>th</sup> pouring, how much alcohol was left in bottle A ?

2. 圖一顯示  $\triangle ABC$ ， $P$  為  $AB$  的中點及  $Q$  是  $CP$  上的一點。已知  $BQ \perp CP$ ， $PQ = 6$  cm、 $CQ = 9$  cm 及  $AQ = 13$  cm。求  $\triangle ABC$  的面積。

Figure 1 shows  $\triangle ABC$ ,  $P$  is the mid-point of  $AB$  and  $Q$  is a point on  $CP$ . It is known that  $BQ \perp CP$ ,  $PQ = 6$  cm,  $CQ = 9$  cm and  $AQ = 13$  cm. Find the area of  $\triangle ABC$ .



圖一

Figure 1

3. 考慮數列  $a_1, a_2, a_3, \dots$ 。定義  $S_n = a_1 + a_2 + \dots + a_n$  其中  $n$  為任何整數。若  $S_n = 2 - a_n - \frac{1}{2^{n-1}}$ ，求  $a_{2016}$  的值。

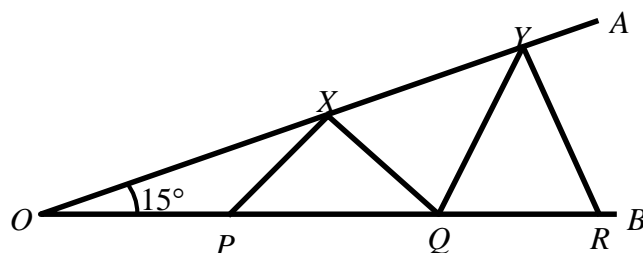
Consider a sequence of numbers  $a_1, a_2, a_3, \dots$ . Define  $S_n = a_1 + a_2 + \dots + a_n$  for any positive integer  $n$ . Find the value of  $a_{2016}$  if  $S_n = 2 - a_n - \frac{1}{2^{n-1}}$ .

4. 設  $x$  及  $y$  為正整數且滿足  $\log x + \log y = \log(2x - y) + 1$ ，求  $(x, y)$  的數量。

If  $x$  and  $y$  are positive integers that satisfy  $\log x + \log y = \log(2x - y) + 1$ , find the number of possible pairs of  $(x, y)$ .

5. 圖二中， $\angle AOB = 15^\circ$ 。X、Y 是 OA 上的點，P、Q、R 是 OB 上的點使得  $OP = 1$  及  $OR = 3$ 。若  $s = PX + XQ + QY + YR$ ，求  $s$  的最小值。

In Figure 2,  $\angle AOB = 15^\circ$ , X, Y are points on OA, P, Q, R are points on OB such that  $OP = 1$  and  $OR = 3$ . If  $s = PX + XQ + QY + YR$ , find the least value of  $s$ .



圖二

Figure 2

6. 設  $y = px^2 + qx + r$  為一二次函數。已知

(1)  $y$  的對稱軸為  $x = 2016$

(2) 該函數的圖像通過  $x$  軸於 A、B 兩點，其中  $AB = 4$  單位

(3) 該函數的圖像通過直線  $y = -10$  於 C、D 兩點，其中  $CD = 16$  單位

求  $q$  的值。

Let  $y = px^2 + qx + r$  be a quadratic function. It is known that

(1) The axis of symmetry of  $y$  is  $x = 2016$ .

(2) The curve cuts the  $x$ -axis at two points A and B such that  $AB = 4$  units.

(3) The curve cuts the line  $y = -10$  at two points C and D such that  $CD = 16$  units.

Find the value of  $q$ .

7. 設三角形三條中線的長度為 9、12 及 15。求該三角形的面積。

The lengths of the three medians of a triangle are 9, 12 and 15. Find the area of the triangle.

8. 若某正整數的二進位表示有以下特質：

(1) 有 11 個位，

(2) 有六個位是 1，有五個位是零，

則稱該數為「好數」。

(例如：2016 是一個「好數」，因為  $2016 = 11111100000_2$ 。)

求所有「好數」的和。

If the binary representation of a positive integer has the following properties:

(1) the number of digits = 11,

(2) the number of 1's = 6 and the number of 0's = 5,

then the number is said to be a “good number”.

(For example, 2016 is a “good number” as  $2016 = 11111100000_2$ .)

Find the sum of all “good numbers”.

9. 設整數  $a$ 、 $b$  及  $c$  為三角形的邊長。已知  $f(x) = x(x-a)(x-b)(x-c)$ ，且  $x$  為一個大於  $a$ 、 $b$  及  $c$  的整數。若  $x = (x-a) + (x-b) + (x-c)$  及  $f(x) = 900$ ，求該三角形三條垂高的總和。

Let the three sides of a triangle are of lengths  $a$ ,  $b$  and  $c$  where all of them are integers. Given that  $f(x) = x(x-a)(x-b)(x-c)$  where  $x$  is an integer of size greater than  $a$ ,  $b$  and  $c$ . If

$x = (x-a) + (x-b) + (x-c)$  and  $f(x) = 900$ , find the sum of the lengths of the three altitudes of this triangle.

10. 求  $\frac{1^4 + 2015^4 + 2016^4}{1^2 + 2015^2 + 2016^2}$  的值。

Find the value of  $\frac{1^4 + 2015^4 + 2016^4}{1^2 + 2015^2 + 2016^2}$ .

完  
END